Solving Linear Equations with the Function Analyzer

In the following exercises, you use technology to reveal the connection between symbolic and graphic representations of equation solving. In these challenges, you'll use an interactive tool called the Function Analyzer available at http://seeingmath.concord.org/resources_files/FunctionAnalyzer.html.

Before You Start

Take a few minutes to become acquainted with the Function Analyzer.

The Function Analyzer provides a virtual model of common algebra manipulatives (algebra tiles), in which one or more bars representing the variable change in area as the value of x changes.

Experiment with the Function Analyzer and try using it to:

- Add positive and negative x-bars, half-x-bars, and constant units, then take them away—noting how your actions are reflected in the symbolic and graphic representations (equation and graph).
- Modify components of the symbolic representation, including the value of the function (the y-value)—noting this time how your actions are reflected in the virtual manipulative and graphic representations.
- Move the slide bar to change the value of x—noting the accompanying change in the tiles, as well as the equation and graph.

When you are satisfied you understand the Function Analyzer, use it to try the challenges below.

CHALLENGES WITH A SINGLE EQUATION

Keep these questions in mind as you work through the first three challenges:

- Do the graphic or area-model representations help you understand:
    - Solving for x?
    - Evaluating for y?
    - The meaning of unknown vs. variable?
- Equivalent functions are functions that have the same graph. Equivalent equations are equations that have the same solution set. (The two meanings of "equivalent" are quite different.) How does your understanding of these concepts help you understand the process of solving a linear equation?
- As you move from one challenge to the next, what changes in the equations? What stays the same?

Challenge 1: Solving the Original Equation

Solve the equation $3x + 2 = 8$ graphically, using the Function Analyzer. Experiment as much as you like.

What steps did you take to arrive at a value for x? What does the interactive show you about the process of solving the equation?

Challenge 2: Solving after the First Operation

Keep the equation $3x + 2 = 8$ in the Function Analyzer.

If you were solving this equation traditionally, your first step would probably be to subtract 2 from each side of the equation, yielding a new equation. Create this new equation, $3x = 6$, in the Function Analyzer and solve it graphically.

Challenge 3: Solving after the Second Operation

Keep the previous two equations in the Function Analyzer.

Solving symbolically, the next step would be to divide both sides of $3x = 6$ by 3. Create another new equation, $x = 2$, in the Function Analyzer and solve that.

CHALLENGES WITH SIMULTANEOUS EQUATIONS

Now you are going to explore the intersection of functions in an entirely different way—as the solution to systems of linear equations. The following exercises are intended to help you explore ways in which the intersections of lines can assist in the solving of equations.

Keep these questions in mind as you work through the next three challenges:

- Problems like these are usually referred to as solving simultaneous *equations*. What do you think about this terminology? Does it influence whether you think of x as an unknown or a variable?
- When you use the Function Analyzer, how would you describe the area model for the two equations at the intersections of their lines? Are there other points at which you could describe the area models in the same way? Explain your answer.
- How does the Function Analyzer's graphic representation help you see differences in the meaning of:
  - finding points that give equivalent values of a function
  - solving an equation
  - finding the x-intercept

Challenge 4: Simultaneous A

Consider the simultaneous linear equations:

$$\begin{cases} y = 0.5x + 1 \\ y = 1.5x - 5 \end{cases}$$

Solve the above system with the Function Analyzer

Challenge 5: Simultaneous B

Consider this system and solve it with the Function Analyzer:

$$\begin{cases} 2y + 3 = 4x + 7 \\ y = 1 + 2x \end{cases}$$

Challenge 6: Simultaneous C

Remember the equation $3x + 2 = 8$ from Challenges 1–3? Using the Function Analyzer, how might you represent this single equation as two simultaneous equations (functions)? What about $3x = 6$? Or $x = 2$? What did this challenge highlight about the process of solving *any* equation?